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A *Touch-Tone*® Receiver-Generator With Digital Channel Filters

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A Touch-Tone® *receiver with cyclotomic digital channel filters introduced in a companion paper is presented in this paper. A comparison with standard digital channel filters reveals that the number of additions per second needed to implement the channel filters is significantly reduced using cyclotomic filters. The performance of cyclotomic filters as a function of their period is presented in graphic form. The results presented here simulating the filter with random inputs indicates that the filters can effectively reject non-*Touch-Tone *signals. Sensitivity of some important criteria as a function of the accuracy of the clock used to control the digital filters is summarized. The results show that the filters are not particularly sensitive to nonaccurate clocks.*

I. INTRODUCTION

In Ref. 1 we describe a family of filters with several advantages over existing filters, which can be used to generate and detect single tones. Here, we describe how such filters can be used in the construction of a *Touch-Tone*® receiver.

The standard *Touch-Tone* receiver is described in Ref. 2; many other receivers have been proposed in the literature; one which is completely digital is described in Refs. 3 and 4, and an analog receiver with a digitally controlled center frequency is described in Ref. 5. The basic *Touch-Tone* telephone must generate tones to identify the ten basic possible inputs (1, 2, $\cdots$, 9, 0) or, in the case of augmented telephones, 12 to 16 possible inputs (including, for example, * and #). This is done by arranging the input buttons in a grid of four rows and three or four columns. Associated with each row is one of four "low" frequencies (697, 770, 852, or 941 Hz), and associated with each column is one of three or four high frequencies (1209, 1336, 1477, or 1633 Hz). When a button is pushed, one low and one high frequency are simultaneously generated, correspondiug to the row and column in which the button is situated. In the central office, a detector decodes the incoming pair of tones to determine which button was pushed.

An incoming signal first passes through a series of tuned filters that filter out dial tones, ring tones, busy tones, and power harmonics (which have amplitude too large to be accommodated by the subsequent channel filters). Next, the signal passes through two parallel bandpass filters (BPF) (see Fig. 1), one to reject the four high-frequency tones (low BPF) and one to reject the four low-frequency tones (high BPF). The output of each BPF passes through a limiter, and the limited signal passes through four parallel channel filters. Each channel filter is connected to a threshold detector which, in 40 ms, makes a determination of whether the tone was present or absent.

In analog receivers, the most critical section consists of the channel filters. Hence, these have to be made with precision components to meet the specifications for station sets. Use of a completely digital receiver requires analog-to-digital (A-to-D) conversion, and special care has to be taken to avoid problems caused by roundoff errors in the BPFs. Furthermore, use of the receiver to generate *Touch-Tone* signals leads to unwanted limit cycles, impairing performance (see Ref. 6).

We propose here a hybrid receiver based on the cyclotomic filters presented in Ref. 1. In the hybrid receiver, the filters that attenuate the dial tone, etc., are the standard analog filters which, using RC active circuitry, can be integrated.[6] The digital part of the receiver follows the limiting circuits (see Fig. 1), which in this case are hard-clippers, thus eliminating the need for separate A-to-D conversion, and at the same time replacing a significant portion of the receiver by digital circuitry. The analog part need not be made with precision components, since variation in the gains of the bandpass filters does not affect the output of the hard limiter significantly. Only the sign of the outputs of the BPFs are used in the digital part of the receiver. The digital filters in the receiver are all operated with perfect arithmetic. All channels have identical filters operating on samples of the output of the hard limiters. However, for each channel, the sampling frequency is proportional to the channel frequency.

Some important features of the system can be summarized as follows:

(*i*) Compared to the channel filters in the all-digital receiver presented in Ref. 3, the number of additions needed to detect tones is relatively small. Hence, fewer adders are needed.

(*ii*) All digital channel filters are mechanized with perfect arithmetic, thus avoiding problems of roundoff.

(*iii*) Since we use perfect arithmetic, we can also generate *Touch-Tone* receiver frequencies using the same channel filters in the receiver.

(*iv*) Without using any A-to-D conversion, we use digital channel filters with analog BPFs.

(*v*) By resetting the filters periodically, we lessen the chance that noise during inter-digit silences, or residual ring tone signals before the first digit, will affect performance.

(*vi*) Since the channel filters have infinite Q, it is possible to increase the signaling rate.

(*vii*) Although the filters have infinite Qs, the peak-to-threshold rejection is kept below 3 dB, thus still preserving the guard action of the hard limiters.

We assume that the reader is familiar with Refs. 1 and 2. Section II gives a description of the hybrid receiver. Section III deals with the performance of the channel filters. Some remarks concerning the factors that enter into choosing the period of cyclotomic filters and interval of operation are contained in Section IV.

II. DESCRIPTION OF THE HYBRID RECEIVER

Figure 1 is a block diagram of the general receiver. The structure of the hybrid receiver is very similar to the standard receiver which is described in Ref. 2. The analog part of the receiver includes both the BPFs and the filters which attenuate power harmonics, ring tones, etc. The outputs of each BPF go into hard-limiters, which convert the analog output of the BPFs into a signal which is either $+1$ or -1, depending

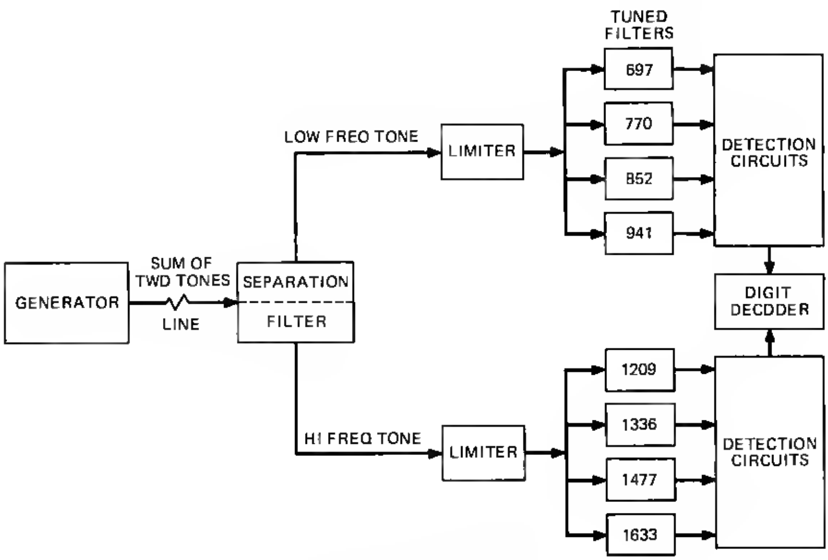

Fig. 1—General receiver.

on whether the analog signal is nonnegative or negative, respectively. This entire analog part can be integrated using active RC circuitry (see Ref. 4). The channel filters which follow the hard-limiters (see Fig. 1) are identical cyclotomic filters (see Ref. 1 and Fig. 2). The cyclotomic filter for each channel has as its input the output of the hard-limiter sampled at a rate p times the channel frequency, where p is the period of the cyclotomic filter used. This requires clock pulses of different frequencies for the different channels.

The channel filters are run periodically for an interval of time inversely proportional to the channel frequency, called the interval of operation. At the beginning of each such interval, the filters are set to zero. The magnitude of the output of each of the filters is compared with a fixed threshold; when the magnitude exceeds this level, a tone corresponding to this frequency is assumed to be present (during the entire interval of operation). The length of the interval of operation is dependent on the permissible error. An interval of operation corresponding to seven cycles of the channel frequency was found to be sufficient (see Section 3.2). This corresponds to 10 ms for the channel corresponding to the lowest *Touch-Tone* frequency, 697 Hz. Hence, if the 697-Hz channel tone is present for the required 40 ms (Ref. 2, p. 11), then in at least three consecutive intervals the tone will produce a signal above the threshold. For higher frequencies, the interval of

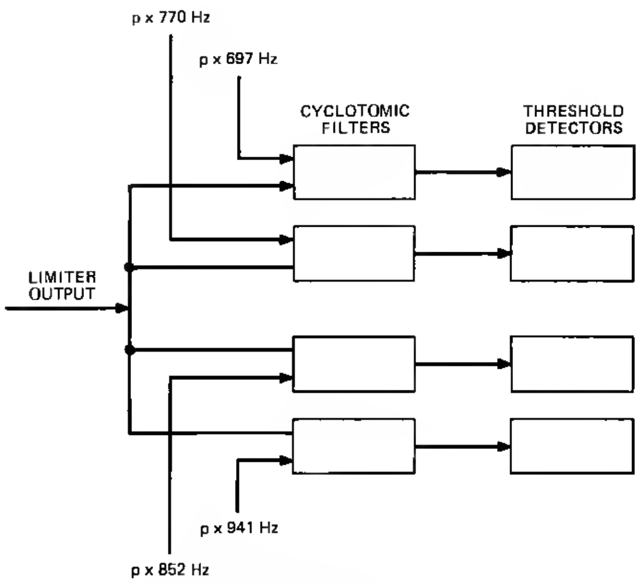

Fig. 2—Channel filters of the low group.

operation is shorter. By synchronizing the intervals of operation of all channels, testing is made for the simultaneous presence of a high tone and a low tone. When a high tone and a low tone are each present for three consecutive intervals, a valid *Touch-Tone* signal is assumed to be present. The digit corresponding to a pair of tones is decoded in the standard way, as described in Section I. Modification of the elementary decision process could be made to increase the signal rate, since the interchannel rejection achieved in a single operating interval is sufficient (see below).

We will not be concerned here with details of hardware in the mechanization of the receiver, but will describe some ways in which the computations in the channel filters can be performed in a multiplexed system.

Two basic modes of implementation will be discussed. One involves individual channel filters dedicated to a fixed frequency. These could be multiplexed to receive inputs from many sources (Fig. 3). This may be more useful in central office applications, where a substantial number of *Touch-Tone* receivers have to be operating at the same time. In this case, the channels controlled by the same clock can be multiplexed in the usual way using serial arithmetic as described in Ref. 1. A system of 20 receivers would require eight clocks (or clock pulses derived from a simple high-frequency clock). For a system using, for example, six times the channel frequency as sampling rate, one adder per channel seems adequate. From Table II, Ref. 1, computations show that the cyclotomic polynomial of period 6, F_6, needs 84 adds per period. The channel corresponding to the highest frequency, 1633 Hz, will need $(1633 \times 84 \times 20)$ adds/s. This implies that an add must not take more than about 0.36 μs. So, with 0.36-μs adders, eight adders would be needed for the whole system. This is, of course, excluding the logic involved in the decision process. If in the system we allow for buffers in the higher frequency channels, then a slower adder could be used, since we wait 10 ms before a decision is made. In this case, the speed of the adder is determined by the channel corresponding to the

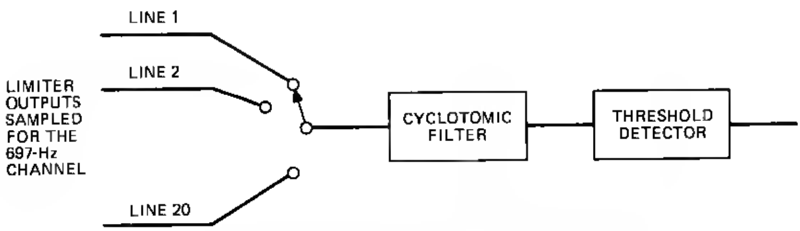

Fig. 3—System amenable to serial multiplexing.

lowest frequency. The lowest frequency channel requires $(697 \times 84 \times 20)$ adds/s, corresponding to an add in $\sim 0.85\ \mu s$.

Another system involves buffering the input in such a way that a single filter can be used for two or more channels (Fig. 4). This might prove useful when an adder is mutliplexed between channels corresponding to the same receiver. In this case, buffers for each channel store the output from the limiter in segments corresponding to the seven-cycle interval of operation. For the filter based upon F_6, this would be 42 bits long. Since the buffer corresponding to a higher frequency would fill up faster than one corresponding to a lower frequency, the channel corresponding to the highest frequency, i.e., 1633 Hz, is fed into the filter first, say, after 5 ms (the buffer of this channel fills up in less than 5 ms). After completing the operation on all the 42 bits of input of this channel, the filter is multiplexed to operate on the next highest frequency channel, and so on. This requires that the adder be fast enough to do 7×84 adds in less than $\frac{5}{8}$ ms, i.e., 940,800 adds/s so a 1-μs adder would suffice. Since this adder is idle for every 5 ms of the 10-ms cycle, it can be used for another receiver. Hence, a 1-μs adder could do all the additions for the channel filters of two receivers. Modification of this elementary decision process could be made depending on the statistics of noise in the channel and sensitivity of the limiter. When a high and a low tone

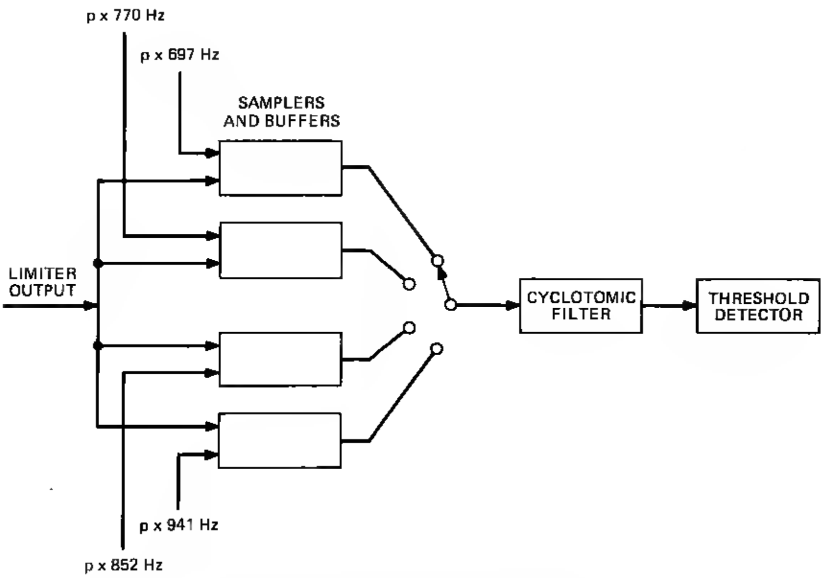

Fig. 4—Multiplexing using buffers.

have been simultaneously detected for three consecutive 10-ms periods, then a decision is made that a *Touch-Tone* signal has been received and the digit is identified in the usual way. The regular second-order filter used in the all-digital receiver of Ref. 3 requires a minimum of 2400 adds/ms and a total of 96,000 adds and achieves an interchannel rejection of $\sim$ 7 dB. Using a cyclotomic filter of period 6 (based on F_6) would require 840 adds to give the same interchannel rejection. This corresponds to a rate up to 60 adds/ms for the 697-Hz channel. If the period were raised to 30 and no use of read-only memory were made, it would still only require a maximum of 56,700 adds to achieve the same rejection; this corresponds to approximately 4010 adds/ms. Of course, intermediate periods would give intermediate statistics, which can be readily computed for systems based on F_p (p = 8, 9, 12, 15, 16, 18, 24; see Ref. 1).

III. PERFORMANCE OF THE CHANNEL FILTERS

To discuss the performance of the channel filters, we need to define certain terms. Let $f_i, i = 1, 2, \cdots, 8$ be the eight channel frequencies. As described earlier, each channel filter is a cyclotomic filter of some period p, based on the cyclotomic polynomial F_p. The order of the filter is denoted by k (the degree of F_p). The fundamental resonance frequency of each filter is determined by τ_i, the sampling interval in seconds of the output of the hard-limiter. In order that the fundamental resonance of the filter be at frequency f_i, τ_i should satisfy

$$p\tau_i = \frac{1}{f_i}.$$

From Ref. 1 we see that the operation of any channel filter can be modeled by

$$x_n = \sum_{i=1}^{k} a_i x_{n-i} + u_n$$

$$y_n = \sum_{i=1}^{k} c_i x_{n-i} \qquad n = 0, 1, \cdots, N$$

$$x_j = 0 \qquad \text{for} \quad j < 0,$$

where x_{n-i}, $i = 1, \cdots, k$ are the numbers stored in the shift register implementing the particular channel filter, y_n the output of the filter, and u_n the sampled output of the hard-limiter, which is, of course, the input to the filter. Hence, if the output of the BPF is a sinusoid of frequency f,

$$\begin{aligned} u_n &= 1 \qquad &\text{if} \quad \sin 2\pi n f\tau \geqq 0 \\ &= -1 \qquad &\text{if} \quad \sin 2\pi n f\tau < 0, \end{aligned}$$

where τ is the sampling interval associated with the channel. So that we may use the same threshold for all channels, we normalize the interval of operation by the fundamental resonant frequency. Hence, if each filter is operated for N steps, this corresponds to operating the filter for $N\tau_i$ s. N/p describes the same interval in units corresponding to a period of the fundamental resonant frequency, hence, an interval of operation of seven periods of the fundamental, i.e., $7 \cdot 1/f_i$ s. We will compare performance of cyclotomic filters of different periods operating for the same number of periods of the fundamental.

Let $M(f)$ denote the maximum absolute value of y_n in the interval of operation when the input square wave is of frequency f. Detection of the fundamental frequency is based on $M(f)$ exceeding a preassigned threshold. A plot of $M(f)$ vs frequency for various cylotomic filters when operated for seven periods of the fundamental is given in Ref. 1. The curves serve to indicate how well the filter performs in distinguishing between tones. The model of a typical curve is shown in Fig. 5. Following standard terminology, we use the term power gain or gain at f to mean $20 \log_{10} M(f)$. Difference between power gains at two frequencies is related in the obvious way to the ratio of $M^2(f)$ at these two frequencies. By scaling the frequency axis linearly, the fundamental resonant frequency can be shifted arbitrarily. The specifica-

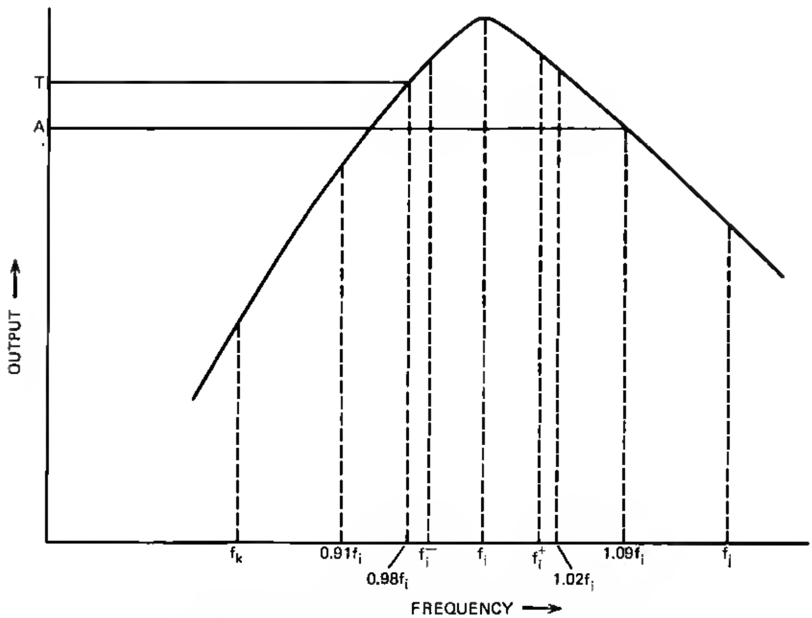

Fig. 5—Specifications for a typical channel.

tions (see Ref. 2, p. 11) for the *Touch-Tone* receiver require that any tone of frequency f lying in the interval I_i defined by $f_i^- \equiv 0.987 f_i - 4 \leqq f \leqq 1.013 f_i + 4 \equiv f_i^+$ be accepted as a tone corresponding to frequency f_i. This band of frequencies is referred to as the accept band of channel i. The threshold T_i has to be set such that $M(f) > T_i$ for all f in the accept band. Therefore, $T_i \leqq \mathrm{Min}_{f_i^- \leq f \leq f_i^+} M(f)$. We call $20 \log_{10} T_i$ the "maximum threshold" for channel i. On the other hand, T_i has to be greater than $M(f)$ for $f \in I_j$, $j \neq i$. We call $A_i \equiv [\mathrm{Max}_{j \neq i} M(f_j)]$ the "maximum gain at reject channels." If the gain at any other channel j exceeds T_i, then a tone corresponding to channel j could be mistaken as one corresponding to i. The threshold with 3-dB rejection is merely $20 \log_{10} A_i + 3$. Use of this threshold assures that if the input to channel i is a signal corresponding to some other channel, then the signal level in the filter is at least 3 dB below threshold. Finally, the "rejection at edge" is the measure of the maximum drop in signal level at the edge frequencies f_i^- and f_i^+ from the center frequency f_i.

Evidently, these parameters are different for different channels. However, by setting certain standards for a typical threshold and maximum reject channel gain, a worst-case standard set for the whole receiver can be found to compare the performance of cyclotomic filters of different periods. It is easily seen that I_i is contained in the interval $[0.98 f_i, 1.02 f_i]$; on the other hand, this interval is not significantly bigger than I_j for any j. For each channel frequency f_i, every $f_j j \neq i$ lies outside the interval $[0.91 f_i, 1.09 f_i]$. The rejection of every alien channel is greater than the rejection of frequencies at ends of this interval because of the bell-shaped nature of the curve in the intervals of interest.

Now that the ends of the intervals of interest have been scaled with respect to the resonant frequency, we can define

$$T = \mathrm{Min}\,[M(0.98 f_i), M(1.02 f_i)]$$
$$A = \mathrm{Max}\,[M(0.91 f_i), M(1.09 f_i)].$$

Then $20 \log_{10} T$ and $20 \log_{10} A$ serve as standards for threshold and maximum reject channel gain for all channels. Figure 6* is a plot of $M(f_i)$, T, and A for cyclotomic filters of periods 6 through 30, run for seven periods of the fundamental resonant frequency. Although the T and A as a percentage of $M(f_i)$ do not change appreciably as the period of the filter increases, the effect of increasing the period of the cyclotomic filter is not equivalent to scaling the input to the filter.

* In Fig. 6, ○, +, and □ correspond to $M(f_i)$, T, and A adjusted for phase shift of input signal as described in Section 3.2.

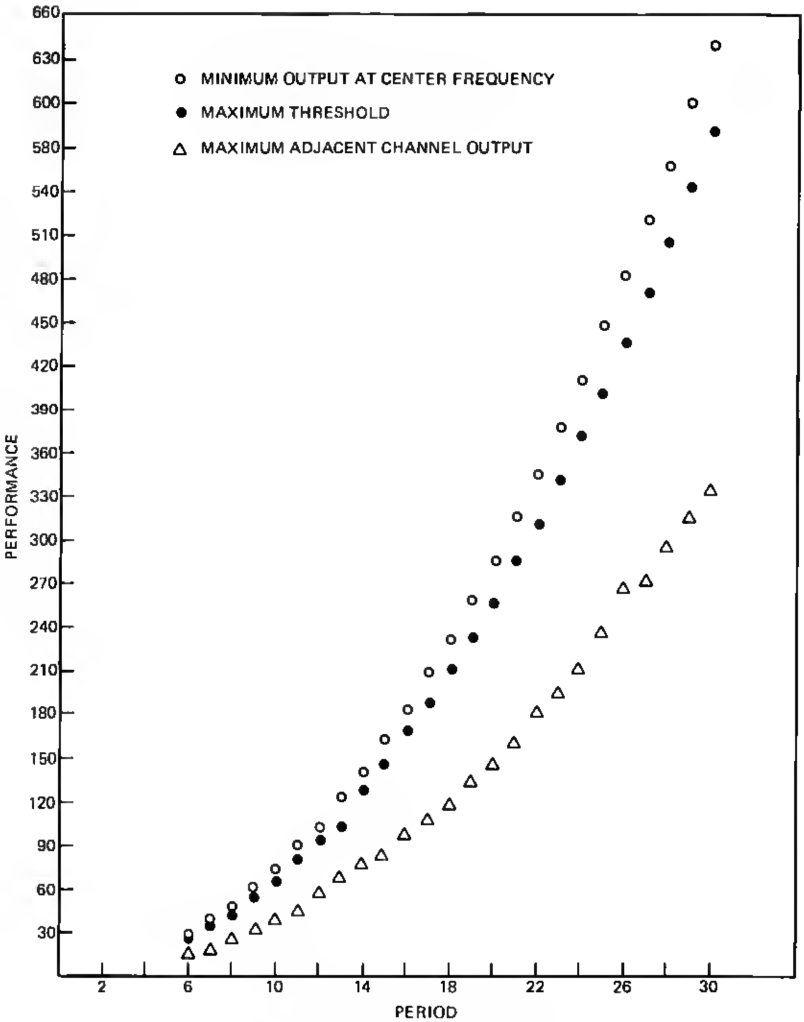

Fig. 6—Performance vs period.

This is because filters of larger periods assume a larger number of distinct levels. Furthermore, increasing the period of the filter may be a way of reducing the effect of noise at the limiter as described in Ref. 1. Although rejection in decibels is a conventional method of describing performance of the tuned filter, the actual level of the signal may be more pertinent to digital applications; hence, the plot is a linear scale. We can now discuss some specific aspects of the performance of these filters.

3.1 Higher-order resonances of filters

Since the channel filters are discrete-time filters, spurious resonances could affect performance, especially since the inputs are hard-clipped, and hence have all odd harmonics (see Ref. 1, Section III). The higher harmonics introduced resulting from clipping could interfere with the fundamental. However, for a cyclotomic filter with transversal weighting function (see Ref. 1) of period p, the spurious resonance closest to the fundamental resonant frequency is $(p - 1)$ times the fundamental. Hence, for example, for $p = 6$ (the lowest period considered here) the closest spurious resonance is five times the fundamental. Therefore, for the channel corresponding to the lowest *Touch-Tone* frequency (697 Hz), the first spurious resonance occurs at 3485 Hz, well outside the *Touch-Tone* band. The higher the period of the cylotomic filter, the further away this resonance will move.

3.2 Interchennel rejection

It was observed above that the ratio $T_i/A_j i \neq j$ was greater than T/A for all channels. Hence, the minimum interchannel rejection is greater than $20 \log_{10} (T/A)$. We will use $20 \log_{10} (T/A)$ as a measure of interchannel rejection. The interchannel rejection for all filters of periods between 6 and 30 varies between 4.2 and 4.9 dB. This is predicated on the assumption that the tone was synchronized with the switching on of the filter. This, of course, need not be the case in practice. Hence, this figure was adjusted for the worst-case phase difference between switching on of the receiver and zero of the time signal. Calculations showed that in all cases the rejection was not lowered by more than 0.5 dB for all filters. The values shown in Fig. 5 are corrected for worst-case phase difference. By increasing the interval of operation to 10 periods of the fundamental, the minimum interchannel rejection for all channels can be increased to about 7 dB. If the interval of operation is of the form $(m + \frac{1}{2})$ periods of the fundamental for any integer m, no correction for phase shift seems to be necessary.

3.3 Sensitivity to clock rate

Some important parameters of the filters corresponding to each channel as a function of percentage variation in sampling rate was calculated. The results when cyclotomic filters of period 6 are used for seven cycles of channel frequency show that with a threshold set at 28 dB above the unit signal level of the hard-clipper, a $\pm$2-percent change in sampling rate can be tolerated. Hence, even though we have to use eight different clock pulses, these clock pulses do not have to be controlled especially accurately. For cyclotomic filters of period 30,

the largest period considered in Ref. 1, similar observations can he made based on computational results. It can be deduced then that the performance of the channel filters are not especially sensitive to clock rate. This allows for the use of cheaper clocks, when each channel is clocked separately.

IV. REJECTION OF PSEUDO *TOUCH-TONE* SIGNALS

Whenever the input to the hard-limiter is a sinusoid, $M(f)$ gives an indication of the signal level in the filter. However, when no *Touch-Tone* signal is present, the output of the BPFs are not sinusoids. Owing to the nonlinear nature of hard-limiting, the curve on Fig. 5 does not lend significant insight into the signal level for complex signals. To simulate a family of non-*Touch-Tone* receiver inputs to the filter, we modeled the output of the hard-limiters as a two-state symmetric Markov chain such that the average numher of changes of sign in the interval of operation was equal to the number of changes of sign of a tone corresponding to the channel frequency. Then a simulation of the filter operating on such inputs was made. The noise level was ahout 12 dB helow the level in the accept hand for all cyclotomic filters of periods 6 through 30.

V. SOME REMARKS ON THE CHOICE OF INTERVAL OF OPERATION AND PERIOD OF CYCLOTOMIC FILTERS USED

As mentioned earlier, an interval of operation corresponding to seven periods is sufficient to provide adequate interchannel rejection. Hence, for signaling it is possihle that a 20-ms on-time requirement for tones might be sufficient. In this case, one can eliminate the need for bandpass filters by altering the signaling process somewhat. Instead of transmitting two tones simultaneously for 40 ms, the tones can be sent one after the other, each heing 20 ms at present. However, it would be necessary to determine whether this scheme can provide adequate speech immunity. This would reduce the number of channel filters to four, since only one frequency from the two groups of frequencies is present at a time. Because of simplifications effected in the receiver, this method of signaling might prove more useful for transmitting information using *Touch-Tone* signaling.

As for the period of the cyclotomic filter used, it is clear from Table II, Ref. 1, that the number of adds/s increases as the period increases. However, depending on the signal-to-noise ratio at the input to the hard limiter, the use of a period high enough to make the frequency of errors in detection small might be necessary.

VI. ACKNOWLEDGMENTS

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